# CH:4 INTEGRATION

## * Standard Integrals:

1) $\int 1.dx = x$

2) $\int K.dx = K.x$

3) $\int \cos x.dx = \sin x$

4) $\int \sin x.dx = -\cos x$

5) $\int \sec^2 x.dx = \tan x$

6) $\int \operatorname{cosec}^2 x.dx = -\cot x$

7) $\int \sec x.\tan x.dx = \sec x$

8) $\int \operatorname{cosec} x.\cot x.dx = -\operatorname{cosec} x$

9) $\int a^x.dx = \frac{a^x}{\log a}$

10) $\int x^n.dx = \frac{x^{n+1}}{n+1}$

11) $\int e^x.dx = e^x$

12) $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$

13) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x$
$= -\cos^{-1} x$

14) $\int \frac{1}{1+x^2} dx = \tan^{-1} x$
$= -\cot^{-1} x$

15) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x$
$= -\operatorname{cosec}^{-1} x$

16) $\int \frac{1}{x} dx = \log|x|$

17) $\int \tan x.dx = \log|\sec x|$

18) $\int \cot x.dx = \log|\sin x|$

19) $\int \sec x.dx = \log|\sec x + \tan x|$
$= \log\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + c$

20) $\int \operatorname{cosec} x.dx = \log|\operatorname{cosec} x - \cot x|$
$= \log\left|\tan\left(\frac{x}{2}\right)\right|$

21) $\int \frac{f'(x)}{f(x)} dx = \log|f(x)|$

21-a) $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)}$

22) $\int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

23) $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|$

24) $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a}.\log\left|\frac{x-a}{x+a}\right|$

25) $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right)$

26) $\int \frac{1}{\sqrt{x^2+a^2}} dx = \log\left|x+\sqrt{x^2+a^2}\right|$

27) $\int \frac{1}{\sqrt{x^2-a^2}} dx = \log\left|x+\sqrt{x^2-a^2}\right|$

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28) $\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}.\sec^{-1}\left(\frac{x}{a}\right)+c$

29) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

30) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c$

31) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}.\sqrt{x^2-a^2}-\frac{a^2}{2}.\log\left|x+\sqrt{x^2-a^2}\right|+c$

* If $\int f(x).\,dx = \phi(x)+c$ then
$\int f(ax+b)\,dx = \frac{1}{a}.\phi(ax+b)+c$

* Useful techniques:

1) Substitution method.

2) $\int \frac{a\sin x+b\cos x}{c\sin x+d\cos x}\,dx$ & $\int \frac{ae^x+b}{ce^x+d}\,dx$

$\Rightarrow$ Put, Numerator = A (Denominator) + B. $\frac{d}{dx}$ (Denominator)

3) $\int \frac{1}{ax^2+bx+c}\,dx$ & $\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx$

$\Rightarrow$ - Make coeff. of $x^2=1$
- Make complete square by adding mid-term
- Use formulae (21 to 27)

4) $\int \frac{px+q}{ax^2+bx+c}\,dx$ & $\int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx$

$\Rightarrow$ (i) ~~px+q = A~~ $N^r = A.\frac{d}{dx}(D^r)+B$
(ii) Find A & B

5) $\int \frac{1}{a+b\sin^2 x}\,dx$ & $\int \frac{1}{a+b\cos^2 x}\,dx$

& $\int \frac{1}{a\sin^2 x + b\cos^2 x + c}\,dx$

⇒ (i) Divide $N^r$ & $D^r$ by $\cos^2 x$ or $\sin^2 x$.
(ii) Replace $\sec^2 x$ / $\mathrm{cosec}^2 x$ by $(1+\tan^2 x)$ / $(1+\cot^2 x)$.
(iii) Put $\tan x = t$ or $\cot x = t$
(iv) Now integral becomes $\int \frac{1}{at^2+bt+c}\,dt$
(v) Now solve like method (3).

6) $\int \frac{1}{a+b\sin x}\,dx$ & $\int \frac{1}{a+b\cos x}\,dx$ & $\int \frac{1}{a\sin x + b\cos x}\,dx$

& $\int \frac{1}{a+b\sin x + c\cos x}\,dx$

⇒ (i) Put $\tan\left(\frac{x}{2}\right) = t$ $\quad \therefore dx = \frac{2\,dt}{1+t^2}$

$\sin x = \frac{2\tan(x/2)}{1+\tan^2(x/2)} = \frac{2t}{1+t^2}$

$\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)} = \frac{1-t^2}{1+t^2}$

- $\int \frac{1}{a+b\sin 2x}\,dx$, $\int \frac{1}{a+b\cos 2x}\,dx$, $\int \frac{1}{a\sin 2x + b\cos 2x}\,dx$

$\int \frac{1}{a+b\sin 2x + c\cos 2x}\,dx$

⇒ (i) Put $\tan x = t$ $\quad \therefore dx = \frac{dt}{1+t^2}$

$\sin 2x = \frac{2t}{1+t^2}$, $\cos 2x = \frac{1-t^2}{1+t^2}$

7) Integration by parts:
$\int u.v\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$

Order of first function:

L I A T E

Logarithmic, Inverse Trigonometric, Algebric, Trigonometric, Exponential.

8) $\int e^x[f(x)+f'(x)]\,dx = e^x.f(x)+C$

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9) Substitutions :

| Expression | Substitution |
|---|---|
| i) $\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$ | $x = a\cos 2\theta$ |
| ii) $\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$ | $x = a^2\cos 2\theta$ |
| iii) $x^2+a^2$ | $x = a\tan\theta$ |
| iv) $a^2-x^2$ | $x = a\sin\theta$ |
| v) $x^2-a^2$ | $x = a\sec\theta$ |

10) $\int \sqrt{a^2-x^2}$, $\sqrt{x^2+a^2}$, $\sqrt{x^2-a^2}$

$\Rightarrow$ Use formulae 29 to 31.

11) $\int (px+q)\sqrt{ax^2+bx+c}\,dx$

$\Rightarrow$ (i) $px+q = A\cdot \frac{d}{dx}(ax^2+bx+c) + B$

(ii) Find A & B

12) Integration by partial fractions:

For $\frac{f(x)}{g(x)}$, Check degree of both

1) If degree of $f(x) <$ degree of $g(x)$

then $\frac{f(x)}{g(x)}$ is proper rational function.

2) If degree of $f(x) >$ degree of $g(x)$

then perform actual division.

$$\frac{f(x)}{g(x)} = \text{Quotient} + \underbrace{\frac{\text{Remainder}}{g(x)}}_{\text{Proper rational function}}$$

| | Rational form | Partial Form |
|---|---|---|
| Type-I i) | $\frac{px+q}{(x-a)(x-b)}$ | $\frac{A}{x-a} + \frac{B}{x-b}$ |
| Type-I ii) | $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$ | $\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ |
| Type-II iii) | $\frac{px+q}{(x-a)^2}$ | $\frac{A}{x-a} + \frac{B}{(x-a)^2}$ |

| | | |
|---|---|---|
| iv) | $\dfrac{px^2+qx+r}{(x-a)^2(x-b)}$ | $\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}+\dfrac{C}{x-b}$ |
| v) | $\dfrac{px^2+qx+r}{(x-a)^3(x-b)}$ | $\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}+\dfrac{C}{(x-a)^3}+\dfrac{D}{x-b}$ |
| vi) | $\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ | $\dfrac{A}{x-a}+\dfrac{Bx+C}{x^2+bx+C}$ |

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① $\int \left[\frac{d}{dx} f(x)\right] dx =$ ______

a) $f'(x)+c$  b) $f(x)+c$  c) $\int f(x) \cdot dx$  d) $e^{f(x)}+c$

→ (b) If $\frac{d}{dx} f(x) = F(x)$ then $\int F(x)\,dx = f(x)+c$

② $\int x(x+1)(x+2)\,dx =$ ______

a) $x^4+x^3+x^2+c$  b) $\frac{x^4}{4}+\frac{x^3}{2}+\frac{x^2}{2}+c$

c) $x^4+x^3+\frac{x^2}{2}+c$  d) $\frac{x^4}{4}+x^3+x^2+c$

→ (d) Let $I = \int x(x+1)(x+2)\,dx$

$= \int (x^2+x)(x+2)\,dx = \int (x^3+2x^2+x^2+2x)\,dx$

$= \int (x^3+3x^2+2x)\,dx$

$= \frac{x^4}{4}+\frac{3x^3}{3}+\frac{2x^2}{2}+c = \frac{x^4}{4}+x^3+x^2+C$

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(3) $\int \frac{x+4}{x+1}\,dx =$ ____

a) $3x+\log(x+1)+c$  b) $x-5\log(x+4)+c$
c) $x+3\log(x+1)+c$  d) $x^3+\log(x+1)+c$

$\Rightarrow$ (c) Let $I = \int \frac{(x+1)+3}{x+1}\,dx$

$= \int \frac{x+1}{x+1}\,dx + \int \frac{3}{x+1}\,dx$

$= \int dx + \int \frac{3}{x+1}\,dx$

$= x + 3.\log(x+1)+c$

(4) $\int \frac{x}{2x+3}\,dx =$ ____

a) $\frac{2}{x}+\frac{4}{3}\log x+c$  b) $\frac{x^2}{2}+\frac{x^2}{6}+c$
c) $\frac{x}{2}-\frac{3}{4}\log(2x+3)+c$  d) $\frac{x}{2}+\frac{4}{3}\log(2x+3)+c$

$\Rightarrow$ (c) Let $I = \frac{1}{2}\int \frac{2x}{2x+3}\,dx = \frac{1}{2}\int \frac{(2x+3)-3}{2x+3}\,dx$

$= \frac{1}{2}\int \left[1-\frac{3}{2x+3}\right]dx$

$= \frac{1}{2}\left[x-3\frac{\log(2x+3)}{2}\right]+c$

$= \frac{x}{2}-\frac{3}{4}.\log(2x+3)+c$

(5) If $\int \frac{3x^2+4x-2}{x-1}\,dx = \frac{3x^2}{2}+ax+b\log(x-1)+c$

then $(a,b) \equiv$ ____
a) $(7,5)$  b) $(5,7)$  c) $(-7,-5)$  d) $(-5,-7)$

$\Rightarrow$ (a) Let $I = \int \frac{3x^2+4x-2}{x-1}$

$$
\begin{array}{r|l}
x-1 \,) \; 3x^2+4x-2 & (\, 3x+7 \\
-\,3x^2-3x & \\
\hline
7x-2 & \\
-\,7x-7 & \\
\hline
5 &
\end{array}
$$

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$$I = \int \left[ (3x+7) + \frac{5}{x-1} \right] dx$$

$$= \frac{3x^2}{2} + 7x + 5 \log(x-1) + c$$

$\therefore a = 7,\ b = 5 \qquad \therefore (a,b) \equiv (7,5)$

6) $\int (2x+1)\sqrt{x+1}\, dx =$ ______

a) $\frac{4}{5}(x+1)^{3/2} - \frac{2}{3}(x+1)^{5/2} + c$ b) $\frac{4}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$

c) $\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$ d) $\frac{5}{2}(x+1)^{2/5} + \frac{3}{2}(x+1)^{2/3} + c$

$\Rightarrow$ (b) let $I = \int (2x+1)\sqrt{x+1}\, dx$

put $x+1 = t \qquad \therefore x = t-1$

$\therefore dx = dt$

$$\therefore I = \int [2(t-1)+1]\sqrt{t}\, dt$$

$$= \int (2t-2+1)\sqrt{t}\, dt = \int (2t-1)\, t^{1/2}\, dt$$

$$= \int (2t^{3/2} - t^{1/2})\, dt$$

$$= 2 \cdot \frac{t^{5/2}}{5/2} - \frac{t^{3/2}}{3/2} + c$$

$$= \frac{4}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$$

7) $\int \frac{1}{x\sqrt{x-1}}\, dx =$ ______

a) $2\cot^{-1}\sqrt{x-1} + c$ b) $2\tan^{-1}\sqrt{x-1} + c$

c) $\sec^{-1} x + c$ d) $\sec^{-1}(\sqrt{x}) + c$

$\Rightarrow$ (b) put $t = \sqrt{x-1}$

$\therefore t^2 = x - 1$

$\therefore x = t^2 + 1$

$\therefore dx = 2t \cdot dt$

$$\therefore I = \int \frac{1}{(1+t^2)\cdot t} \cdot 2t\, dt = 2\int \frac{1}{1+t^2}\, dt$$

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$= 2\tan^{-1} t + c$

$= 2\tan^{-1}(\sqrt{x-1}) + c$

8) $\int (\sin^4 x - \cos^4 x)\,dx =$ ____

a) $\sin x - \cos x + c$ b) $\sin x \cdot \cos x + C$

c) $-\sin x \cdot \cos x + c$ d) $\cos 2x + C$

$\Rightarrow$ (c) $I = \int (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)\,dx$

$= \int 1 (\sin^2 x - \cos^2 x)\,dx$

$= \int -\cos 2x \cdot dx$

$= -\dfrac{\sin 2x}{2} + c = -\dfrac{2\sin x \cdot \cos x}{2} + c$

$= -\sin x \cdot \cos x + c$

9) $\int \dfrac{\cos 2x}{\sin^2 x \cdot \cos^2 x}\,dx =$ ____

a) $\tan x - \cot x$ b) $\cot x - \tan x$

c) $\tan x + \cot x$ d) $-(\tan x + \cot x)$

$\Rightarrow$ (d) $I = \int \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x \cdot \cos^2 x}\,dx$

$= \int \left[\dfrac{1}{\sin^2 x} - \dfrac{1}{\cos^2 x}\right] dx$

$= \int (\operatorname{cosec}^2 x - \sec^2 x)\,dx$

$= -\cot x - \tan x + c$

$= -(\tan x + \cot x) + C$

10) $\int \cot^2\left(\dfrac{x}{3}\right) dx =$ ____

a) $x + 3\cot(x/3)$ b) $x - 3\cot(x/3)$

c) $x + 3\cot x$ d) $-x - 3\cot(x/3)$

$\Rightarrow$ (d) $I = \int \left[\operatorname{cosec}^2\left(\dfrac{x}{3}\right) - 1\right] dx$

$$= \frac{-\cot(x/3)}{1/3} - x = -x - 3\cot(x/3) + C$$

11) $\int (\sec x + \tan x)^2 dx =$ ____

a) $x + \sec x + \tan x$ b) $2(\sec x + \tan x) - x$

c) $2(\sec x - \tan x) + x$ d) $2(\sec x + \tan x) + x$

$\Rightarrow$ (b) $I = \int (\sec^2 x + 2\sec x.\tan x + \tan^2 x)\, dx$

$$= \int (\sec^2 x + 2\sec x.\tan x + \sec^2 x - 1)\, dx$$

$$= \int (2\sec^2 x + 2\sec x.\tan x - 1)\, dx$$

$$= 2\tan x + 2\sec x - x + C$$

$$= 2(\tan x + \sec x) - x + C$$

12) $\int \frac{1}{x + x^{-3}} dx =$ ____

a) $\tan^{-1}(x^2) + c$ b) $\log(x + x^{-3}) + c$

c) $\frac{-1}{1+x^4} + c$ d) $0.25 \log(1+x^4) + c$

$\Rightarrow$ (d) $I = \int \frac{1}{x + \frac{1}{x^3}} dx = \int \frac{x^3}{x^4+1} dx$

$$= \frac{1}{4}\int \frac{4x^3}{x^4+1} dx \qquad \left\{ \int \frac{f'(x)}{f(x)} dx = \log[f(x)] \right.$$

$$= \frac{1}{4}.\log|x^4+1| + c$$

$$= 0.25 \log(1+x^4) + c$$

13) $\int \frac{e^{2x}+1}{e^{2x}-1} dx =$

a) $\log(e^x - e^{-x}) + \alpha$ b) $\log(e^x + e^{-x}) + \beta$

c) $\log(e^{2x}-1) + x + \gamma$ d) $\log(e^{2x}-1) + \delta$

$\Rightarrow$ (a) $I = \int \frac{e^x + 1/e^x}{e^x - 1/e^x} .dx \qquad \left\{ \text{Divide } N^r \,\&\, D^r \text{ by } e^x \right.$

$$= \int \frac{e^x + e^{-x}}{e^x - e^{-x}} .dx$$

$$= \log(e^x - e^{-x}) + \alpha \qquad \left\{ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| \right.$$

14) $\int \frac{\tan^{-1}x}{1+x^2}\,dx =$ ____

a) $\log(\tan^{-1}x)+c$ b) $0.5(\tan^{-1}x)^2+c$

c) $\log(1+x^2)+c$ d) $\tan^{-1}(\log x)+c$

$\Rightarrow$ (b) Put $\tan^{-1}x = t$

$\therefore \frac{1}{1+x^2}\,dx = dt$

$\therefore I = \int t\cdot dt = \frac{t^2}{2} = \frac{1}{2}(\tan^{-1}x)^2+c$

$= 0.5(\tan^{-1}x)^2+c$

15) $\int\sqrt{\frac{1+\cos 3x}{1-\cos 3x}}\,dx =$ ____

a) $\cot\left(\frac{3x}{2}\right)+c$ b) $\tan\left(\frac{3x}{2}\right)+c$

c) $\frac{2}{3}\log\left[\sin\left(\frac{3x}{2}\right)\right]+c$ d) $3\log(\sin 3x)+c$

$\Rightarrow$ (c) $I = \int\sqrt{\frac{1+\cos 3x}{1-\cos 3x}\times\frac{1+\cos 3x}{1+\cos 3x}}\,dx$

$= \int \frac{1+\cos 3x}{\sqrt{1-\cos^2 3x}}\,dx = \int \frac{1+\cos 3x}{\sin 3x}\,dx$

$= \int \frac{2\cos^2(3x/2)}{2\sin\left(\frac{3x}{2}\right)\cdot\cos\left(\frac{3x}{2}\right)}\,dx$

$= \int \cot\left(\frac{3x}{2}\right)dx$

$= \frac{\log\left|\sin\left(\frac{3x}{2}\right)\right|}{3/2} = \frac{2}{3}\log\left[\sin\left(\frac{3x}{2}\right)\right]+c$

16) If $\int(\tan^2x+\tan^4x)\,dx = \left(\frac{1}{n}\right)\tan^n x+c$, then $n =$ ____

a) 2 b) 3 c) 4 d) 5

$\Rightarrow$ (b) $I = \int \tan^2x\cdot(1+\tan^2x)\cdot dx$

$= \int \tan^2x\cdot\sec^2x\cdot dx$

put $\tan x = t$

$\therefore \sec^2x\cdot dx = dt$

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$\therefore I = \int t^2 \cdot dt = \frac{t^3}{3} + C$

$= \frac{1}{3} \tan^3 x + C$

Comparing with given integral, $\left(\frac{1}{n}\right) \tan^n x + C$
we get, $n = 3$

17) $\int \frac{1}{x^2+6x+10} dx =$ ____

a) $\frac{1}{2} \cdot \log \left[\frac{x+3}{x-3}\right] + c$ b) $\sin^{-1}(x+3) + C$

c) $\log(x + \sqrt{x+3}) + C$ d) $\tan^{-1}(x+3) + C$

$\Rightarrow$ (d) $I = \int \frac{1}{(x^2+6x+9)+1} dx$

$= \int \frac{1}{1+(x+3)^2} dx = \tan^{-1}(x+3) + C$

18) $\int \frac{3}{4-5\sin x} dx =$ ____

a) $\log \left[\frac{\tan(x/2)-2}{\tan(x/2)-1}\right] + c$ b) $\log \left[\frac{2\tan x - 4}{2\tan x - 1}\right] + c$

c) $\log \left[\frac{2\tan(x/2)-4}{2\tan(x/2)-1}\right] + c$ d) $\tan^{-1}\left[2\tan\frac{x}{2} - 5\right] + c$

$\Rightarrow$ (c) put $\tan\left(\frac{x}{2}\right) = t$ $\therefore dx = \frac{2}{1+t^2} dt$

& $\sin x = \frac{2t}{1+t^2}$

$\therefore I = \int \frac{3}{4 - 5 \cdot \frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2} \cdot dt$

$= \int \frac{6}{4(1+t^2)-10t} \cdot dt$

$= \int \frac{6}{4+4t^2-10t} \cdot dt = \frac{6}{4} \int \frac{1}{t^2 - \frac{5}{2}t + 1} \cdot dt$

$= \frac{3}{2} \int \frac{1}{\left(t^2 - \frac{5}{2}t + \frac{25}{16}\right) + 1 - \frac{25}{16}} dt$

$= \frac{3}{2} \int \frac{1}{\left(t - \frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2} \cdot dt$

$= \frac{3}{2} \times \frac{1}{2 \times \frac{3}{4}} \cdot \log \left| \frac{t - \frac{5}{4} - \frac{3}{4}}{t - \frac{5}{4} + \frac{3}{4}} \right| = \log \left| \frac{4t-8}{4t-2} \right|$

$$= \log\left|\frac{3t-4}{2t-1}\right| = \log\left|\frac{3\tan(x/2)-4}{2\tan(x/2)-1}\right|$$

19) $\int \frac{\cos x}{\sqrt{17-\cos^2 x}}\,dx =$ ____

a) $\log(\sin x - \sqrt{17-\cos^2 x}) + c$ b) $\log(\sin x + \sqrt{16+\sin^2 x}) + c$
c) $\log(\sqrt{17-\cos^2 x} - \sqrt{1-\cos^2 x}) + c$
d) $4\tan^{-1}(2\cos x) + c$

$\Rightarrow$ (b) $I = \int \frac{\cos x}{\sqrt{17-(1-\sin^2 x)}}\,dx = \int \frac{\cos x}{\sqrt{16+\sin^2 x}}\,dx$

Put $\sin x = t$
$\therefore \cos x \cdot dx = dt$

$\therefore I = \int \frac{1}{\sqrt{16+t^2}}\,dt = \int \frac{1}{\sqrt{t^2+(4)^2}}\,dt$

$= \log|t + \sqrt{t^2+16}| + c$
$= \log|\sin x + \sqrt{16+\sin^2 x}| + c$

20) If $\int \frac{3e^x+4e^{-x}}{5e^x+6e^{-x}}\,dx = ax + b\cdot\log(5e^{2x}+6) + c$
then ____

a) $a = 2/3,\ b = -30$ b) $a = 3/2,\ b = -1/30$
c) $a = 2/3,\ b = -1/30$ d) $a = -3/2,\ b = -30$

$\Rightarrow$ (c) $I = \int \frac{3e^x + 4/e^x}{5e^x + 6/e^x}\,dx = \int \frac{3e^{2x}+4}{5e^{2x}+6}\,dx$

let $3e^{2x}+4 = A(5e^{2x}+6) + B\left[\frac{d}{dx}(5e^{2x}+6)\right]$

$= A(5e^{2x}+6) + B(10e^{2x})$

$\therefore 3e^{2x}+4 = (5A+10B)e^{2x} + 6A$

Equating coeff.

$5A+10B = 3$, $6A = 4$

$\therefore 5\left(\frac{2}{3}\right)+10B = 3$ $\therefore \underline{A = 2/3}$

$\therefore \frac{10}{3} + 10B = 3$

$\therefore 10B = 3 - \frac{10}{3} = -\frac{1}{3}$

$\therefore \underline{B = -\frac{1}{30}}$

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$$\therefore I = \int \frac{\frac{2}{3}(5e^{2x}+6) - \frac{1}{30}(10e^{2x})}{5e^{2x}+6}\,.dx$$

$$= \frac{2}{3}\int 1.dx - \frac{1}{30}\int \frac{10e^{2x}}{5e^{2x}+6}\,dx$$

$$= \frac{2}{3}x - \frac{1}{30}\log(5e^{2x}+6)+C$$ → Comparing with given eqⁿ.

$$\therefore a = \frac{2}{3},\quad b = -\frac{1}{30}$$

21) If $\int (x+1)\sqrt{2x-1}\,dx = a(2x-1)^{5/2} + b(2x-1)^{3/2} + C$ then $b/a =$ ____

a) 6 b) 5 c) 4 d) 3

⇒ (b) let $I = \int (x+1)\sqrt{2x-1}\,dx$

put $2x-1=t$ $\therefore 2x = t+1$ $\therefore x = \frac{t+1}{2}$

$\therefore 2dx = dt$

$\therefore dx = \frac{1}{2}dt$

$$\therefore I = \int \left(\frac{t+1}{2}+1\right).\sqrt{t}.\frac{1}{2}dt$$

$$= \frac{1}{2}\int \left(\frac{t+3}{2}\right).t^{1/2}.dt = \frac{1}{2}\int\left[\frac{t^{3/2}}{2} + \frac{3t^{1/2}}{2}\right]dt$$

$$= \frac{1}{4}.\frac{t^{5/2}}{5/2} + \frac{3}{4}.\frac{t^{3/2}}{3/2} + C$$

$$= \frac{2}{20}.(2x-1)^{5/2} + \frac{6}{12}(2x-1)^{3/2} + C$$

$$= \frac{1}{10}(2x-1)^{5/2} + \frac{1}{2}(2x-1)^{3/2} + C$$

Comparing, we get $a = \frac{1}{10}$, $b = \frac{1}{2}$

$$\therefore \frac{b}{a} = \frac{1/2}{1/10} = \frac{10}{2} = 5$$

22) $\int e^x.\frac{\sin x - \cos x}{\sin^2 x}.dx =$ ____

a) $e^x.\sec x + c$ b) $e^x.\csc x + C$

c) $e^x.\tan x + c$ d) $e^x.\cot x + C$

⇒ (b) $I = \int e^x\left[\frac{\sin x}{\sin^2 x} - \frac{\cos x}{\sin^2 x}\right]dx$

$$= \int e^x\left[\frac{1}{\sin x} - \frac{1}{\sin x}\,\frac{\cos x}{\sin x}\right]dx$$

$= \int e^x (\operatorname{cosec} x - \operatorname{cosec} x . \cot x) dx$

$= e^x . \operatorname{cosec} x + c$ $\qquad \therefore \int e^x [f x + f'(x)] = e^x . f(x) + c$

23) $\int x . \sin x . dx =$ ____

a) $\sin x - x \cos x + c$ b) $\cos x - x \sin x + c$

c) $1 - \cos x + c$ d) Does not exist.

$\Rightarrow$ (a) Let $I = \int x . \sin x \, dx$

~~$= \int \sin x \, dx \, dx$~~ LIATE

$= x \int \sin x \, dx - \int \left[\frac{d}{dx} x . \int \sin x . dx \right] dx$

$= x . (-\cos x) - \int 1 . (-\cos x) dx$

$= -x \cos x + \int \cos x . dx$

$= -x \cos x + \sin x + c$

$= \sin x - x . \cos x + c$

24) $\int \log x . dx =$ ____

a) $\frac{1}{x} + c$ b) $x . \log \left(\frac{x}{e}\right) + c$ c) $x . \log \left(\frac{e}{x}\right) + c$

d) $e^{\log x} + c$

$\Rightarrow$ (b) $I = \int \log x . 1 . dx$

$= \log x \int 1 . dx - \int \left[\frac{d}{dx} \log x . \int 1 . dx\right] dx$

$= \log x . x - \int \frac{1}{x} . x . dx$

$= x \log x - x + c$

$= x \log x - x . \log e + c$ $\qquad [\because \log e = 1]$

$= x . \log \left(\frac{x}{e}\right) + c$

25) $2 \int \sec^3 x . dx =$ ____

a) $\sec x . \tan x + \log (\sec x + \tan x) + c$

b) $\csc x . \cot x - \log (\csc x - \cot x) + c$

c) $\sec x . \tan x - \log (\sec x + \tan x) + c$

d) $\csc x . \cot x + \log (\csc x + \cot x) + c$

$\Rightarrow$ (a) $I = 2 \int \sec x . \sec^2 x . dx$

$= 2\sec x \int \sec^2 x \cdot dx - 2 \int \left( \frac{d}{dx} \sec x \int \sec^2 x \, dx \right) dx$

$= 2\sec x \cdot \tan x - 2 \int \sec x \cdot \tan x \cdot \tan x \cdot dx$

$= 2\sec x \cdot \tan x - 2 \int \sec x \tan^2 x \cdot dx$

$= 2\sec x \cdot \tan x - 2 \int \sec x ( \sec^2 x - 1) \, dx$

$= 2\sec x \cdot \tan x - 2 \int \sec^3 x \, dx + 2 \int \sec x \, dx$

$= 2\sec x \cdot \tan x - I + 2 \log(\sec x + \tan x) + c$

$\therefore 2I = 2\sec x \cdot \tan x + 2 \log(\sec x + \tan x) + c$

$\therefore I = \sec x \cdot \tan x + \log(\sec x + \tan x) + c$

26) $\int 2^{3x} \cdot 3^{2x} \cdot dx =$ ______

a) $2^{3x} \cdot 3^{2x} \cdot (\log 2)(\log 3) + c$ b) $2^{3x-1} \cdot 3^{2x-1} \cdot (\log 2)(\log 3) + c$
c) $2^{3x} \cdot 3^{2x} \cdot \log 5 + c$ d) $2^{3x} \cdot 3^{2x} \cdot \log_{72} e + c$

$\Rightarrow$ (d) $I = \int 2^{3x} \cdot 3^{2x} \cdot dx$ 100

$= \int (2^3 \cdot 3^2)^x \cdot dx \quad = \int (8 \times 9)^x \, dx$

$= \int 72^x \cdot dx$

$= \dfrac{72^x}{\log_e 72} + c \quad = \dfrac{2^{3x} \cdot 3^{2x}}{\log_e 72} + c$

$= 2^{3x} \cdot 3^{2x} \cdot \log_{72} e + c$

27) $\int \dfrac{\sin(\log x)}{x \cdot \cos(\log x)} \, dx =$ ______

a) $\log(\sec x) + c$ b) $\sec(\log x) + c$
c) $\log[\sec(\log x)] + c$ d) $\log[\log(\sec x)] + c$

$\Rightarrow$ (c) put $\log x = t \quad \therefore \frac{1}{x} dx = dt$

$\therefore I = \int \dfrac{\sin t}{\cos t} \cdot dt = \int \tan t \cdot dt$

$= \log(\sec t) + c = \log[\sec(\log x)] + c$